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AN APPROACH TO SPLINES FROM ALGEBRAIC GEOMETRY

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Resumen. A spline is a function which is conformed by pieces of polynomials defined on a rectilinear partition of a domain in n-dimensional space, and joined together to ensure some degree of global smoothness. When modeling curves and surfaces algebraically, using just one polynomial does not give much flexibility unless the polynomial has a very large degree. This is why it is preferable to use splines to approximate large regions of a Computer Aided Geometric Design (CAGD)-model. Spline functions have become universally recognized as highly effective tools not only in CAGD but also in Approximation Theory, Image Analysis, Computer Graphics, Numerical Analysis and Isogeometric Analysis. For a subdivided domain, the C^r splines involving only polynomials of up to some fixed degree, form a vector space. A fundamental problem in this area is to determine a formula for the dimension of this space, in terms of known information about the subdivision. This problem turns to be a quite difficult issue that has not been solved in full generality.

The approach from algebraic geometry to the dimension problem, and the construction of bases for splines has been developed during the last few years, yielding generalizations of previous results and bringing connections of splines with classical problems.

In this talk, using an approach form homological algebra and exploring connections of splines with ideal generated by powers of linear forms, we establish formulas for lower and upper bounds on the dimension. We will also use a local characterization to study the ring structure of those elements of the Stanley–Reisner ring associated to the domain, which correspond to splines of some order of smoothness. We will present some examples, results and conjectures about the generators of the space of splines as a ring.