SEMINARIO DEPARTAMENTO de MATEMÁTICA
UNIVERSIDAD TÉCNICA FEDERICO SANTA MARÍA
MARTES 12 de Noviembre de 2013, 11.30 am
Sala de Seminarios, Edificio F, UTFSM

ON SOME RECENT RESULTS AND OPEN PROBLEMS IN GEOMETRY OF NUMBERS

NIKOLAY MOSHCHEVITIN

FACULTY OF MECHANICS AND MATHEMATICS,
M. V. LOMONOSOV MOSCOW STATE UNIVERSITY, RUSIA.

We suppose to give a survey lecture on some problems in Geomerty of Numbers and Diophantine approximation. Among these problems we discuss a long-standing conjecture due to J. E. Littlewood. Given $\alpha, \beta \in \mathbb{R}$, Littlewood conjectured that

$$\inf_{q \in \mathbb{Z}_+} q|\sin q\alpha \, \sin q\beta| = 0.$$

This conjecture is very easy to formulate and there were many attempts to solve it, by various methods. However, it is still open. Probably this is the most exiting conjecture related to Geometry of Numbers.

Also we discuss some problems and conjectures formulated by W. M. Schmidt in the 1970s. Some of them were solved recently. One of them is as follows. By easy application of the pigeon hole principle one can show that there exist infinitely many integer triples (m_0, m_1, m_2) with

$$|m_0 + m_1\alpha + m_2\beta| \leq (\max(|m_1|, |m_2|))^{-2}.$$

Schmidt proved that there exist infinitely many integer triples (m_0, m_1, m_2) with *positive* m_1, m_2 and such that

$$|m_0 + m_1\alpha + m_2\beta| \leq (\max(m_1, m_2))^{-\frac{1+\sqrt{5}}{2}}.$$

Then Schmidt conjectured that the exponent $\frac{1+\sqrt{5}}{2}$ may be replaced by $2-\varepsilon$ with arbitrary small ε. Recently it was found out that his conjecture is false and that the exponent $\frac{1+\sqrt{5}}{2}$ here is optimal.

CONTACTO SEMINARIOS: pablo.aguirre@usm.cl. (HTTP://PAGUIRRE.MAT.UTFSM.CL/SEMINARIOS.HTML)