Dynamics of partial discharges involved in electrical tree growth in insulation and its relation with the fractal dimension

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Partial Discharges

- Partial discharge (PD) is a localised electric breakdown of a small portion of electrical insulation, usually due to small defects in insulation such as voids, cracks or inclusions.

- PDs may cause insulation to deteriorate, leading to catastrophic failure of the electrical power equipment.

- Analysis of PD is one of the most important diagnostic tools for the degradation of the insulation of electrical power equipment.

[R. Schurch *Partial discharge energy and electrical tree volume degraded in epoxy resin.* 2015]

Electrical trees

- Electrical trees are generated by PD that cause a slow and progressive deterioration of insulating materials, with the formation of numerous, branching partially conducting discharge channels.

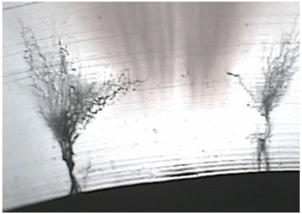


Figure: Electrical tree growing in an insulation.

Fractal dimension of electrical trees

- Electric trees are not fully understood and have complex structure that is not possible to describe analytically.

- By means of the fractal dimension we can classify electrical trees (d_f): Branch type ($1 < d_f < 2$) and Bush type ($2 < d_f < 3$).

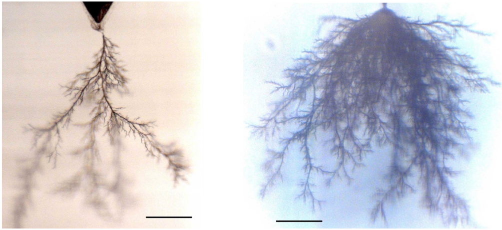


Figure: Branch type (left) and Bush type (right).

Methodology

- Two time series of PD pulses measured during the growth of electrical trees, one at 10kV and the other at 12kV.

- Each sample was stressed for 115 minutes. The resulting trees were 3D scaned using X-ray Computed Tomography.

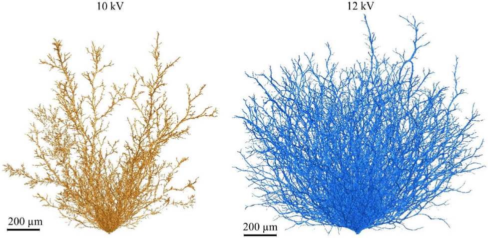


Figure: 3D rendering samples aged at 10kV and 12kV.

- The fractal dimension of the electrical tree aged at 10 kV is 1.89 and the fractal dimension of the tree aged at 12kV is 1.99 using box-counting method.

Methodology

- It was studied the PD with dinamycal systems.
- The phase space it was reconstructed using the delay coordinate embbeding method by Takens.
- The fractal dimension was calculated using correlation dimension and it was related to fractal dimension of electrical trees.

Phase space reconstruction

- The method known as delay coordinate embedding it enables us to construct a phase space of a system from a single observed variable given by the serie $\{x_0, x_1, \ldots, x_N\}$.

- It states that for a large enough embedding dimension m, and an appropriate delay embedding τ the delay vectors

$$p(t) = (x_t, x_{t+\tau}, x_{t+2\tau}, \ldots, x_{t+(m-1)\tau})$$

  yield a phase space with exactly the same invariant quantities as the original system.

[M. Perc *Introducing nonlinear time series analysis in undergraduate courses.* 2004.]

Time Series

- In order to reconstruct the attractor we consider the time serie Q_1 and Q_2 that are the pulse amplitude (Q) of the partial discharges for 10kV and 12kV.

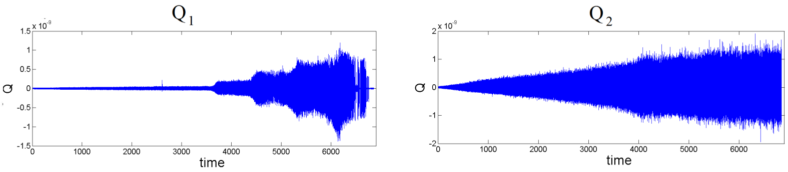


Figure: Time series Q_1,Q_2 for 10kV and 12kV.

Embbedding Delay τ

- A suitable embedding delay τ has to be large enough so that the information we get from measuring the value of variable $x_{t+\tau}$ is different from x_t.
- Fraser and Swinney proposed to use the first minimum of the mutual information between x_t and $x_{t+\tau}$ as the optimal embedding delay.
- Quantifies the amount of information we have about the state $x_{t+\tau}$ presuming we know the state x_t.
- The optimum delay for Q_1 is $\tau = 10$ and for Q_2 is $\tau = 8$ (Using a matlab rutine).

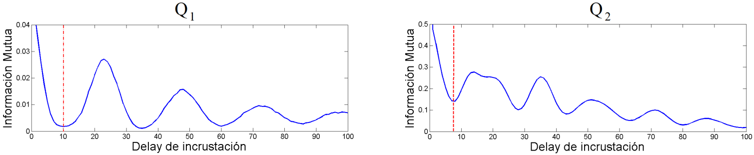


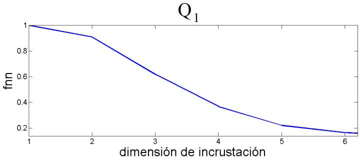
Figure: Primer mínimo en $\tau = 10$ y $\tau = 8$ para Q_1 y Q_2 respectivamente.

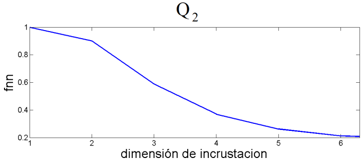
Embedding dimension m

- The false nearest neighbour method (fnn) was introduced by Kennel as an efficient tool for determining the minimal required embedding dimension m.
- The fnn are the points that are close to each other but under short forward iteration the distance grows further than a threshold. The distance between each other is given by

$$R_i = \frac{|x_{i+m\tau} - x_{t+\tau}|}{||p(i) - p(t)||}$$

- We have to minimize the fraction of points having a false nearest neighbour by choosing a sufficiently large m. In our case the embedding dimension is $m = 6$ for a rutine in Matlab.





Test

- The Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories.
- The time series must originate from a deterministic process in order to justify the calculation of the maximal Lyapunov exponent.
- The serie is deterministic, the test proposed by Kaplan and Glass that assures the uniqueness of solutions in the phase space is near 1. The values for Q_1 and Q_2 are

$$q_1 = 0.924 \qquad q_2 = 0.878$$

- The serie originate from a chaotic dynamical system, the maximal lyapunov exponent is positive. The values for Q_1 and Q_2 are

$$\lambda_1 = 0.113 \qquad \lambda_2 = 0.189$$

Strange attractor reconstruction

- Now we can reconstruct the phase space, that corresponds to a strange attractor for both time series Q_1, Q_2. A projection of the phase space is shown in the figure.

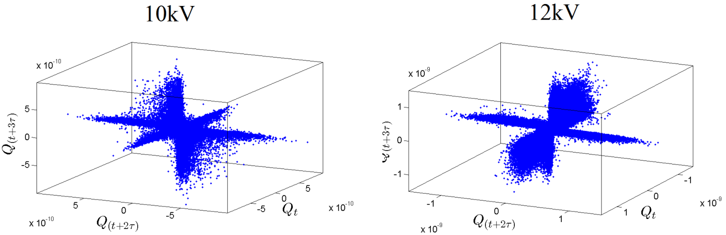


Figure: Projection of phase space for 10kV and 12kV

Fractal dimension of the strange attractor

- Strange attractors are typically characterized by fractal dimensionality D which is smaller than the number of degrees of freedom m.
- The measure is obtained by considering correlations between points of a time series on the attractor.
- The definition of the correlation integral given by Grassberger and Procaccia where Θ is the Heaviside function is

$$C(N,r) = \frac{1}{N(N-1)} \sum_{i \neq j} \Theta(r - ||x_i - x_j||)$$

[J. Theiler *Estimating fractal dimension.* 1989]

Fractal dimension of the strange attractor

- The correlation dimension is

$$\nu = \frac{d \ \log \ C(N,r)}{d \log r}$$

- The dimension of the attractor with 500 points is 1.984 for Q_1 as is 2.303 for Q_2.

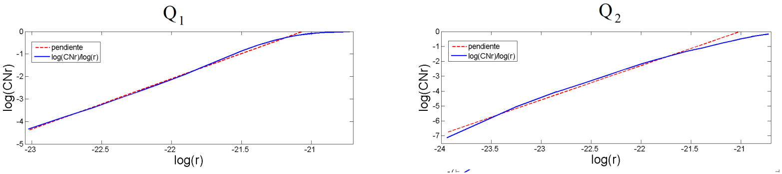


Figure: Slopes of 1.984 for $10kV$ and 2.303 for $12kV$.

Conclusions

- Verify that PD are a deterministic process.
- Reconstruct the strange attractor of the dynamical system associated.
- The dimension of the strange attractor shows a direct correlation to the dimension of the electrical tree associated. These results are not conclusive due to the small amount of data used.

Possible future work:

- Analyse the strange attractor of another data set such as the time when the PDs happens.
- Calculate the fractal dimension of the strange attractor with more data and propose a correlation between the dimensions of the strange attractor and the electrical tree.