

**A PREDATOR-PREY SYSTEM INVOLVING FIVE LIMIT CYCLES.**

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ABSTRACT. In this paper we consider the multiparameter system introduced in [M. Scheffer et al. *Can.J. Fish. Aquat. Sci.* 57(6);1208-1219 (2000)] which corresponds to an extension of the classic minimal Daphnia-algae model. It is shown that there is a neighborhood in the parameter space where the system in the realistic quadrant has a unique equilibrium point which is a repelling weak focus of order four enclosed by a global attractor hyperbolic limit cycle. For a small enough change of the parameters in this neighborhood, bifurcate from the weak focus four infinitesimal Hopf limit cycles (alternating the type of stability) such that the last bifurcated limit cycle is an attractor. Moreover, for certain values of parameters we concluded that this applied model has five concentric limit cycles, three of them being stable hyperbolic limit cycles. This gives a positive answer to a question raised in [C.S. Coleman, *Differential Equations Models*, V 1. 279-297, Springer-Verlag (1978)] and [N.G. Lloyd et al. *Appl. Math. Lett.* Vol 9, No. 1, 15-18 (1996)].

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